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B.Sc-part-I(H), paper – I, Date- 18-04-2020

Set theory

Q.1. Define and explain the terms:
(I) Numerically equivalent sets (II) Finite sets
(III) Denumerable Sets (IV) Countable sets (V) uncountable sets
illustrate them with suitable examples.

Soln: I Numerically equivalent sets:→ If there exists a mapping $f: A \to B$, which is one-one onto, then the two sets A and B are said to be numerically equivalent and written as $A \sim B$.

$A \sim$ is read as 'A wiggle B'

For example; Let $f: A \to B$, where $A = \{1,2,3\}$ and $\{2,4,6\}$.
Here, $f(x) = 2x$, $\forall x \in A$ and one-one correspondence exists between A and B. Also f is an onto mapping, hence the sets A and B are equivalent.

(II) Finite sets:→ A set which consists of a definite number of elements is called finite sets.

For example: $A = \{x : x$ is an integer $-3 < x < 7\}$

Then $A = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$.

III Denumerable Sets:→ If a set X is equinumerous to the set N, then it is said to be denumerable sets.

For example: $X = \{1, \frac{1}{2}, \cdots, \frac{1}{n}, \cdots\}$, $N \approx X$ with $f(n) = \frac{1}{n}$.

(IV) Countable sets:→ Let a set X is equinumerous to the set N then it is said to be countable sets.

For example: $Z = \{0, 1, -1, 2, -2, \cdots, n, -n, \cdots\}$
and $N = \{1, 2, 3, 4, 5, \cdots, n, \cdots\}$

(V) Uncountable sets:→ Let a set is neither finite nor countable, it is said to be uncountable sets.

For example: $I = \{x / 0 < x < 1\}$ is uncountable.
Then since $I \subset R$, the uncountability of R will follow.

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Q.2 Prove that the set Q of all rational numbers is denumerable.

Sol: proof:- Let the set of all positive rationals
$$Q^+ = \{x : x = p/q,\ p, q \in N\}$$
and arrange it in the following form

$A_1 \rightarrow$	$\frac{1}{1} \rightarrow$	$\frac{1}{2}$	$\frac{1}{3} \rightarrow$	$\frac{1}{4}$	$\frac{1}{5} \rightarrow$	$\frac{1}{6}$	------
A_2	$\frac{2}{1}$	$\boxed{\frac{2}{2}}$	$\frac{2}{3}$	$\boxed{\frac{2}{4}}$	$\frac{2}{5}$	$\boxed{\frac{2}{6}}$	------
A_3	$\frac{3}{1}$	$\frac{3}{2}$	$\boxed{\frac{3}{3}}$	$\frac{3}{4}$	$\frac{3}{5}$	$\boxed{\frac{3}{6}}$	------
A_4	$\frac{4}{1}$	$\boxed{\frac{4}{2}}$	$\frac{4}{3}$	$\boxed{\frac{4}{4}}$	$\frac{4}{5}$	$\boxed{\frac{4}{6}}$	------
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Here, A_n stands for all rationals with n as numerator. This table contains all positive rationals. Now arranging them in the order shown by the arrowed line in the above table avoiding repetition, we have
$$\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{3}{1}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{2}, \frac{4}{1}, \ldots\ldots$$
which are to be counted as
$$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, \ldots\ldots$$
Thus the set of positive rationals is countable.

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